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Comparison of Prediction Methods for Damping of a Symmetric Balanced Laminated Composite Beam

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To determine the effective damping of a symmetric, balanced laminated composite, three different analytical models were compared. In the first model, Adams and Ni's theory was used. In the second model, modified classical lamination theory based upon the elastic-viscoelastic correspondence principle was used. In the third model, an energy approach was developed to investigate the damping of laminated composite beams. Four typical laminated composites with $[\pm\theta]$s, $[0/\pm\theta]$s, $[0/\theta]$s and $[0/\pm\theta/90]$s stacking sequences were employed for this study.

Key Words: Vibration, Damping, Loss Factor, Laminated Composites, Classical Lamination Theory, Energy Approach, Force Balance Approach, Elastic-Viscoelastic Correspondence Principle

1. Introduction

Composites have been used to manufacture enclosures, airframe structures, sports goods, helicopter blades, spacecraft and ground vehicles. Composites have more attractive properties such as light weight, superior strength and stiffness. than the conventional materials. The structural elements usually are subject to undesirable vibrations. In order to use composites as dynamic structural members, their vibration damping properties must be understood (Adams, 1987; Hashin, 1970). These damping properties are important factors in the design of composite dynamic structures. Therefore, it is necessary to compare prediction methods and determine the most efficient method for predicting the damping of composites.

Adams and Ni (1984) developed a model to provide designers with a useful and accurate prediction method for damping of composites. Their model predicted damping in laminated composites related to 3 sources of energy dissipation with respect to the in-plane stresses σ_x, σ_y and σ_{xy} in the fibre coordinate system. Sun and coworkers (1987) proposed another damping prediction method which use the classical lamination theory and elastic-viscoelastic correspondence principles. We have further developed and modified these theories.

The objective in this paper is to develop a new mathematical model to predict the damping values of symmetric balanced laminated composites. Material damping usually occurs as the result of the flexural vibration. Thus, we accounted for all possible flexural moments on the laminated composite beam during free vibration. In the proposed damping prediction model, all feasible moments in the constitutive equations of the plate theory of laminated composites are taken into account by considering only the principal curvature (χ_1) corresponding to mode 1 shape (i. e. first flexural mode). The damping of a symmetric balanced laminated composite predicted in this paper is based on the modified classical laminated theory with a complex modulus and the proposed new model. It is compared with that of Adams and Ni's model.

2. Analysis

In this study, three analytical approaches were

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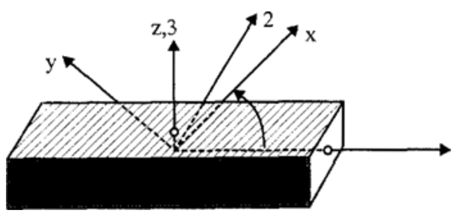
Table 1 Basic material properties of AS4/3501-6 composites used in this study.

E_L (GPa)	E_T (GPa)	ν_{LT}	G_{LT} (GPa)	η_L (x10^{-3})	η_T (x10^{-3})	η_{LT} (x10^{-3})	$\eta_{\upsilon LT}$ (x10^{-3})
128	9.26	0.3	5.9	1.8405	8.5801	9.477	9.204



x : fiber direction
y : normal to the fiber direction
z : normal to x-y plane
1,2 : loading direction

Fig. 1 Fiber and loading coordinate systems used in the analytical models.

employed. The first model devoloped by Adams and Ni is the energy approach. The damping loss factor, η, is defined as ;

$$\eta=\frac{\Delta W}{2\pi W}$$

where ΔW is the energy dissipated during a stress cycle and W is the maximum strain energy. The second model which was modified is to use the force balance approach. In this approach, we obtain the [A], [B] and [D] stiffness matrix in laminated composites by utilizing classical lamination theory. The damping of laminated composites is defined as the ratio of each component of [A], [B] and [D] loss moduli to their storage ones. The final model is a new energy approach. Three analytical formulation is performed on the coordinate system as shown in Fig. 1. The basic material data used for three models are shown in Table 1 as well.

2.1 Modification of basic damping loss factors

In order to calculate the dissipated energy in composites, it is essential to accurately evaluate the basic damping loss factors. In general, it is known that an increase in the amount of damage in the material, the stress amplitude of the test, or the test frequency tends to enhance the damping loss factor.

The theoretical models that are currently available to predict the damping loss factor for composites are inadequate for design purposes. A theoretical model (Hashin, 1970) assumes that adhesives are flexible and the bonding between the fiber and the resin is perfect. However, practically, it is very difficult to fabricate specimens to exactly suit the model. Also, damping can be caused by interface mechanisms such as shearing motion between the fiber and the matrix. Moreover, as the structural dimensions increase, the number of defects in the material increases as well. Thus, discrepancies between theoretical predictions and practical results are unavoidable. In order to make up for this problem, it is proposed that the formula of the basic damping loss factors in laminated composites must include a curve fitting parameter on the nondimensional stiffness terms, based on Hashin's theory (1970) and rule of mixtures.

From an elastic-viscoelastic correspondence principle, Hashin (1970) derived expressions for the basic damping loss factor of a laminated composite. For axial loading, the basic damping property (η_L) is represented by

$$\eta_L=\eta_m(1-V_f)\frac{E_m}{E_L} \tag{1}$$

where, η_L is the axial damping loss factor in composites, η_m is the damping loss factor of matrix, V_f is the fiber volume fraction in composites, E_m is the Young's modulus of matrix, E_L is the axial Young's modulus of composites.

However, these theoretical values do not agree with experimental values. Despite considerable effort to eliminate extraneous losses, it was found (Adams, 1987) that this expression considerably underestimates the experimental value of η_L. Several factors were thought (Adams, 1987) to contribute to the discrepancy : fiber diameter, fiber misalignment, imperfections in the material (cracks and debonds) and the interfacial shear effect. Thus, a curve fitting parameter (α) is herein introduced in the relationship between the axial damping loss factor (η_L) and the stiffness term $(\frac{E_{Lf}}{E_m})$:

$$\eta_L = \frac{\eta_m V_m}{v_m + V_f (\frac{E_{Lf}}{E_m})^{\alpha}} \quad (2)$$

where,

η_m=the damping loss factor of the matrix, η_L= the axial damping loss factor of a unidirectional composites, V_m=the matrix volume fraction, V_f= the fiber volume fraction, E_m=the matrix Young's modulus, E_L=the axial Young's modulus of composites, and α=the curve fitting parameter for axial loading.

Similarly, for longitudinal shear loading, it is proposed that the theoretical equation be modified by introducing the parameter (β), which is fitted to average experimental results by trial and error :

$$\eta_{LT} = \frac{\eta_m (1-v_f)[(G+1)^{(2+\beta)} + V_f (G-1)^{(2+\beta)}]}{[G^{(1+\beta)}(1+V_f)+1-V_f][G^{(1-\beta)}(1-V_f)+1+V_f]} \quad (3)$$

where, $G = G_f/G_m$, G_f=the longitudinal shear modulus of the fiber, Gm=the shear modulus of the matrix, and β=the curve fitting parameter for longitudinal shear loading.

For the transverse damping loss factor (η_T), the rule of mixtures and the elastic-viscoelastic correspondence principle give us the following equation :

$$\eta_T = \eta_m - \frac{\eta_m V_f}{V_f + V_m (\frac{E_{Tf}}{E_m})} \quad (4)$$

This equation may be modified by introducing parameter (ζ) as follows :

$$\eta_T = \eta_m - \frac{\eta_m V_f}{V_f + V_m (\frac{E_{Tf}}{E_m})^{\zeta}} \quad (5)$$

where, η_T=the transverse damping loss factor of a unidirectional composites, E_{Tf}=the transverse Young's modulus of the fiber, and ζ=the curve fitting parameter for transverse loading.

For the Poisson's damping loss factor (η_{vLT}), the rule of mixtures and the elastic-viscoelastic correspondence principle give us the following equation :

$$\eta_{vLT} = \frac{\eta_{vLTf} V_f + \eta_{vm} v_{LTm} V_m}{V_f v_{LTf} + V_m v_{LTm}} \quad (6)$$

where, η_{vLT}=the Poisson's damping loss factor, η_{vLTf}=the Poisson's damping loss factor of the fiber, v_{LTf}=the Poisson's ratio of the fiber, η_{vm}= the Poisson's damping loss factor of the matrix.

Hence, the accuracy of the predicted damping loss factor in any laminated composite depends on the choice of the basic damping experimental data (i. e. η_L, η_T, η_{LT} and η_{vLT} of 0° unidirectional specimen). Technically, it is difficult to accurately locate the resonance frequency and the half-power points due to the limited frequency resolution of the analyzing instrument. Therefore, the basic damping loss factors can only be estimated statistically by trial and error. In addition, a rigorous and standardized experimental method must be developed to obtain consistent damping results in all materials.

2.2 Adams and Ni's approach

In this theoretical analysis (Ni and Adams, 1984), only the principal flexrual moment (M_1) is considered on the specimen. By employing plate theory for laminated composites, the strains in the k^{th} layer are determined. The constitutive equation associated with stresses and strains in the k^{th} layer gives us in-plane stresses in the loading coordinate system. For the beam specimen analysis, σ_2^k and σ_6^k are not considered since σ_1^k is much larger than they. Also, ε_2^k is neglected since ε_2^k is much smaller than ε_1^k and ε_6^k. From Adams and Ni's (1984), the strain energy dissipation, ΔW, which is subjected to bending in the symmetric laminated beam is divided into three parts related to in-plane stresses, ε_x, ε_y and ε_{xy}, in the fiber coordinate system :

$$\Delta W = \Delta W_x + \Delta W_y + \Delta W_{xy} \quad (7)$$

The strain energy dissipation about σ_x is written as

$$\Delta W_x = \int_0^l 2\int_0^{h/2} \pi \eta_L \sigma_x \varepsilon_x dz dx = 2\pi\eta_L \int_0^l \int_0^{h/2} \sigma_x \varepsilon_x dz dx \quad (8)$$

$$\Delta W_x = \frac{2\pi\eta_L}{I^{*2}} \int_0^{h/2} m^2 (Q_{11} d_{11}^* + Q_{12} d_{12}^* + Q_{16} d_{16}^*)$$

$$(m^2 d_{11}^* + mnd_{16}^*) z^2 dz \int_0^l M_1^2 dx$$

where,

l is the length of beam, h is the thickness of beam,

$\eta_L = \dfrac{\eta_m V_m}{V_m + V_f (\frac{E_{Lf}}{E_m})^\alpha}$ is the axial damping loss factor of 0° unidirectional composites,

η_m is the matrix resin damping loss factor,

M_1 is the principal bending moment in the cantilever beam,

α is the curve fitting parameter,

I^* is the normalized moment of inertia,

d_{ij}^* is the normalized flexural compliance component.

Similarly, ΔW_y and ΔW_{xy} can be evaluated as follows :

$$\Delta W = \frac{2\pi\eta_T}{I^{*2}} \int_0^{h/2} n^2 (Q_{11} d_{11}^* + Q_{12} d_{12}^* + Q_{16} d_{16}^*) (n^2 d_{11}^* - mnd_{16}^*) z^2 dz \int_0^l M_1^2 dx \quad (9)$$

$$\Delta W_{xy} = \frac{2\pi\eta_{LT}}{I^{*2}} \int_0^{h/2} mn (Q_{11} d_{11}^* + Q_{12} d_{12}^* + Q_{16} d_{16}^*) (2mnd_{11}^* - (m^2 - n^2) d_{16}^*) z^2 dz \int_0^l M_1^2 dx \quad (10)$$

The bending strain energy of the beam is

$$W_b = \int_0^l M_1 x_1 dx = \frac{d_{11}^*}{I^*} \int_0^l M_1^2 dx \quad (11)$$

The total damping loss factor (η_{ov}) in laminated composites is then expressed as :

$$\psi_{ov} = \frac{\Sigma \Delta W}{\Sigma W} = \frac{\Delta W_x + \Delta W_y + \Delta W_{xy}}{W_b} \quad (12)$$

where,

$\psi_{cv} = 2\pi\eta_{ov}$ is the specific damping capacity in laminated composites, η_{ov} is the overall damping loss factor in laminated composites.

2.3 Modified classical lamination approach

In order to obtain the complex modulus, we relied on the elastic-viscoelastic correspondence principle. The complex equation includes two real constants for storage and loss moduli. The material damping for a symmetric balanced laminated composite is obtained as the ratio of the loss modulus to the storage modulus. Using the elastic-viscoelastic correspondence principle (Sun et. al., 1987), we describe the basic engineering constants in a complex form as follows :

$$E_L^* = E_L (1 + i\eta_L) \quad (13)$$

$$E_T^* = E_T (1 + i\eta_T) \quad (14)$$

$$G_{LT}^* = G_{LT} (1 + i\eta_{LT}) \quad (15)$$

$$v_{LT}^* = v_{LT} (1 + i\eta_{vLT}) \quad (16)$$

where,

$$\eta_L = \frac{\eta_m V_m}{V_m + V_f (\frac{E_{Lf}}{E_m})^\alpha}$$

$$\eta_L = \eta_m - \frac{\eta_m V_f}{V_f + V_m (\frac{E_{Tf}}{E_m})^\zeta}$$

$$\eta_{LT} = \frac{\eta_m (1 - V_f) [(G+1)^{(2+\beta)} + V_f (G-1)^{(2-\beta)}]}{[G^{(1+\beta)}(1+V_f) + 1 - V_f][G^{(1+\beta)}(1-V_f) + 1 + V_f]}$$

$$\eta_{vLT} = \frac{\eta_{vLTf} V_f + \eta_{vm} v_{LTm} V_m}{V_f v_{LTf} + V_m v_{LTm}}$$

in which, $V_f = 1 - V_m$ is the fiber volume fraction, α, ζ and β is the curve fitting parameters for data reduction,

$$G = \frac{G_f}{G_m}.$$

The relations between Q_{ij} and the basic engineering constants are described as :

$$Q_{xx} = \frac{E_L}{1 - v_{LT}^2 (\frac{E_T}{E_L})} \quad (17)$$

$$Q_{xy} = \frac{v_{LT} E_T}{1 - v_{LT}^2 (\frac{E_T}{E_L})} \quad (18)$$

$$Q_{yy} = \frac{E_T}{1 - v_{LT}^2 (\frac{E_T}{E_L})} \quad (19)$$

$$Q_{ss} = G_{LT} \quad (20)$$

Substituting Eqs. (7), (10) into Eqs. (11), (14), we obtained the following relations :

$$Q_{xx}^* = Q'_{xx} + iQ''_{xx} \quad (21)$$

$$Q_{yy}^* = Q'_{yy} + iQ''_{yy} \quad (22)$$

$$Q_{xy}^* = Q'_{xy} + iQ''_{xy} \quad (23)$$

$$Q_{ss}^* = G_{LT}^* = G_{LT} (1 + i\eta_{LT}) \quad (24)$$

To calculate [A], [B] and [D], we transformed [Q] with reference to the fiber orientation of each layer. The transformed $\overline{Q'_{ij}}$ and $\overline{Q''_{ij}}$ (i, j = 1, 2, 6), which are related to Qij (i, j = x, y, s), are described as

$$\overline{Q_{11}} = Q_{xx}\cos^4\theta + Q_{yy}\sin^4\theta + 2(Q_{xy} + 2Q_{ss})\sin^2\theta\cos^2\theta \quad (25)$$

$$\overline{Q_{22}} = Q_{xx}\sin^4\theta + Q_{yy}\cos^4\theta + 2(Q_{xy} + 2Q_{ss})\sin^2\theta\cos^2\theta \quad (26)$$

$$\overline{Q_{12}} = (Q_{xx} + Q_{yy} - 4Q_{ss})\sin^2\theta\cos^2\theta + Q_{xy}(\sin^4\theta + \cos^4\theta) \quad (27)$$

$$\overline{Q_{66}} = (Q_{xx} + Q_{yy} - 2Q_{xy} - 2Q_{ss})\sin^2\theta\cos^2\theta + Q_{ss}(\sin^4\theta + \cos^4\theta) \quad (28)$$

$$\overline{Q_{16}} = (Q_{xx} - Q_{xy} - 2Q_{ss})\sin\theta\cos^3\theta + (Q_{xy} - Q_{yy} - 2Q_{ss})\sin^3\theta\cos\theta \quad (29)$$

$$\overline{Q_{26}} = (Q_{xx} - Q_{xy} - 2Q_{ss})\sin^3\theta\cos\theta + (Q_{xy} - Q_{yy} - 2Q_{ss})\sin\theta\cos^3\theta \quad (30)$$

The expression of A_{ij}, B_{ij} and D_{ij} from the classical lamination theory and the elastic-viscoelastic correspondence principle are given by

$$A_{ij}^* = A'_{ij} + iA''_{ij} = \sum_{k=1}^{N} (\overline{Q'_{ij}} + i\overline{Q''_{ij}})^{(k)}(h_k - h_{k-1}) \quad (31)$$

$$B_{ij}^* = B'_{ij} + iB''_{ij} = \frac{1}{2}\sum_{k=1}^{N} (\overline{Q'_{ij}} + i\overline{Q''_{ij}})^{(k)}(h_k^2 - h_{k-1}^2) \quad (32)$$

$$D_{ij}^* = D'_{ij} + iD''_{ij} = \frac{1}{3}\sum_{k=1}^{N} (\overline{Q'_{ij}} + i\overline{Q''_{ij}})^{(k)}(h_k^3 - h_{k-1}^3) \quad (33)$$

where, $i = \sqrt{-1}$.

We converted these modulus constants obtained from a symmetric balanced laminated composite into the corresponding complex modulus constants using elastic-viscoelastic correspondence principle. We then evaluated the material damping of a symmetric balanced laminated composite as follows :

$$A\eta_{ij}(stretching) = \frac{A''_{ij}}{A'_{ij}} \quad (34)$$

$$B\eta_{ij}(coupling) = \frac{B''_{ij}}{B'_{ij}} \quad (35)$$

$$D\eta_{ij}(bending) = \frac{D''_{ij}}{D'_{ij}} \quad (36)$$

where, i, j = 1, 2, 6.

Consequently, the effective laminate engineering constants E_x, E_y, υ_{xy} and G_{xy} can be expressed as functions of the stretching stiffness constants, A_{ij}, and the laminate thickness. We evaluated the storage stiffness constants A_{ij} and the corresponding dissipated constants, each separately, from the relationships between the effective engineering constants and [A] stiffness constants. We then obtained the material damping for the symmetric balanced laminated composites in terms of the measurable engineering properties for comparison with predictions.

2.4 Proposed energy approach

From the plate theory of symmetric balanced laminated composites we can express the curvature components as follows :

$$\chi_1 = d_{11}M_1 + d_{12}M_2 + d_{16}M_6 \quad (37)$$

$$\chi_2 = d_{12}M_1 + d_{22}M_2 + d_{26}M_6 \quad (38)$$

$$\chi_6 = d_{16}M_1 + d_{26}M_2 + d_{66}M_6 \quad (39)$$

where, d_{ij} is the flexural compliance matrix components of the laminates.

And the relations of moment deflection in the laminated beam are given by

$$M_1 = -D_{11}\frac{\partial^2 w}{\partial x^2} \quad (40)$$

$$M_2 = -D_{12}\frac{\partial^2 w}{\partial x^2} \quad (41)$$

$$M_6 = -D_{16}\frac{\partial^2 w}{\partial x^2} \quad (42)$$

where, D_{ij} is the flexural stiffness matrix components of the laminates, w is the first mode shape.

Accordingly, the curvature components can be rewritten as

$$\chi_1 = -d_{11}D_{11}\frac{\partial^2 w}{\partial x^2} - d_{12}D_{12}\frac{\partial^2 w}{\partial x^2} - d_{16}D_{16}\frac{\partial^2 w}{\partial x^2} \quad (43)$$

$$\chi_2 = -d_{12}D_{11}\frac{\partial^2 w}{\partial x^2} - d_{22}D_{12}\frac{\partial^2 w}{\partial x^2} - d_{26}D_{16}\frac{\partial^2 w}{\partial x^2} \quad (44)$$

$$\chi_6 = -d_{16}D_{11}\frac{\partial^2 w}{\partial x^2} - d_{26}D_{12}\frac{\partial^2 w}{\partial x^2} - d_{66}D_{16}\frac{\partial^2 w}{\partial x^2} \quad (45)$$

Under flexural loading, ply stresses are not linear in a symmetric balanced laminates, but ply strains are linear. During free vibration of a laminated composite specimen, the specimen can be subjected to various flexural moments due to different ply stresses. Thus, we take into consideration only the principal curvature component, χ_1 corresponding to its fundamental bending mode shape. Then, the strains in the k^{th} layer are expressed by

$$\varepsilon_1^k = zk_1 = -z(d_{11}D_{11} + d_{12}D_{12} + d_{16}D_{16})\frac{\partial^2 w}{\partial x^2} \tag{46}$$

where, z is the distance of the k^{th} layer from the midplane.

The stress-strain constitutive equations in the k^{th} layer are

$$\sigma_1^k = -Q_{11}z(d_{11}D_{11} + d_{12}D_{12} + d_{16}D_{16})\frac{\partial^2 w}{\partial x^2} \tag{47}$$

$$\sigma_2^k = -Q_{12}z(d_{11}D_{11} + d_{12}D_{12} + d_{16}D_{16})\frac{\partial^2 w}{\partial x^2} \tag{48}$$

$$\sigma_6^k = -Q_{16}z(d_{11}D_{11} + d_{12}D_{12} + d_{16}D_{16})\frac{\partial^2 w}{\partial x^2} \tag{49}$$

in which Q_{ij} is the stiffness matrix components of k^{th} lamina.

From coordinate transformation relations for the stress, the stresses of a symmetric balanced laminated composites in the fiber coordinate system can be expressed as

$$\sigma_x = (m^2Q_{11} + n^2Q_{12})\left[-z(d_{11}D_{11} + d_{12}D_{12})\frac{\partial^2 w}{\partial x^2}\right] \tag{50}$$

$$\sigma_y = (n^2Q_{11} + m^2Q_{12})\left[-z(d_{11}D_{11} + d_{12}D_{12})\frac{\partial^2 w}{\partial x^2}\right] \tag{51}$$

$$\sigma_{xy} = (-mnQ_{11} + mnQ_{12})\left[-z(d_{11}D_{11} + d_{12}D_{12})\frac{\partial^2 w}{\partial x^2}\right] \tag{52}$$

where, $m = \cos\theta_k$, $n = \sin\theta_k$.

The strains of the k^{th} layer in the fiber coordinate are transformed in a similar approach as follows :

$$\varepsilon_x = -m^2z(d_{11}D_{11} + d_{12}D_{12})\frac{\partial^2 w}{\partial x^2} \tag{53}$$

$$\varepsilon_y = -n^2z(d_{11}D_{11} + d_{12}D_{12})\frac{\partial^2 w}{\partial x^2} \tag{54}$$

$$\varepsilon_{xy} = 2mnz(d_{11}D_{11} + d_{12}D_{12})\frac{\partial^2 w}{\partial x^2} \tag{55}$$

When considering an element of the k^{th} layer of unit width at distance z from the midplane, the total stored energy in the x direction, W_x, can be evaluated by taking the volume integration of strain energy density. The dissipated energy in this layer can be expressed by

$$\Delta W_x = 2\pi\eta_L W_x \tag{56}$$

The energy dissipation, in more detail, is

$$\begin{aligned}\Delta W_x &= 2\pi\int_0^l\int_o^{h/2}\eta_L\sigma_x\varepsilon_x dz dx \\ &= 2\pi\eta_L\int_0^l\int_o^{h/2}\sigma_x\varepsilon_x dz dx = 2\pi\eta_L\int_0^{h/2}m^2 \\ &\quad (d_{11}D_{11} + d_{12}D_{12})^2(m^2Q_{11} + n^2Q_{12})z^2dz \\ &\quad \int_0^l\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x^2}dx\end{aligned} \tag{57}$$

Similarly, ΔW_y and ΔW_{xy} can be evaluated as follows :

$$\begin{aligned}\Delta W_y = 2\pi\eta_T\int_o^{h/2}n^2(d_{11}D_{11} + d_{12}D_{12})^2 \\ (n^2Q_{11} + m^2Q_{12})z^2dz\int_0^l\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x^2}dx\end{aligned} \tag{58}$$

$$\begin{aligned}\Delta W_{xy} = 2\pi\eta_{LT}\int_o^{h/2} -2mn(d_{11}D_{11} + d_{12}D_{12})^2 \\ (-mnQ_{11} + mnQ_{12})z^2dz\int_0^l\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x^2}dx\end{aligned} \tag{59}$$

For the beam, the overall damping η_{ov} is then expressed by dividing the total dissipated energy by the total stored energy as follows :

$$\eta_{ov} = \frac{\sum\Delta W}{2\pi\sum W} = \frac{\eta_L W_x + \eta_T W_y + \eta_{LT}W_{xy}}{W_x + W_y + W_{xy}} \tag{58}$$

3. Results and Discussion

The effective laminate damping loss factors determined by Adams and Ni's theory, the modified classical laminate theory and proposed energy theory are illustrated for the four laminates $[\pm\theta]$s, $[0/\pm\theta]$s, $[0/\theta]$s and $[0/\pm\theta/90]$s in Figs. (2~5). In the case of $[\pm\theta]$s graphite/epoxy laminate, predicted damping obtained from theoretical models indicate a similar trend (Fig. 2). Predicted damping for the other (i. e., $[0/\pm\theta]$s, $[0/\theta]$s and $[0/\pm\theta/90]$s) graphite/epoxy laminates indicate some divergence. For example, In the Adams and Ni's theory, predictions for the $[0/\pm\theta]$s, $[0/\theta]$s and $[0/\pm\theta/90]$s laminate, shown in Figs. (3~5), exhibit a similar trend to those of the off-axis composites. The damping increases to a maximum value in the range of 15° to 30° and decreases slightly as the angle approaches 90°. On the other hand, in the modified classical laminate theory and the proposed theory, the maximum value of damping

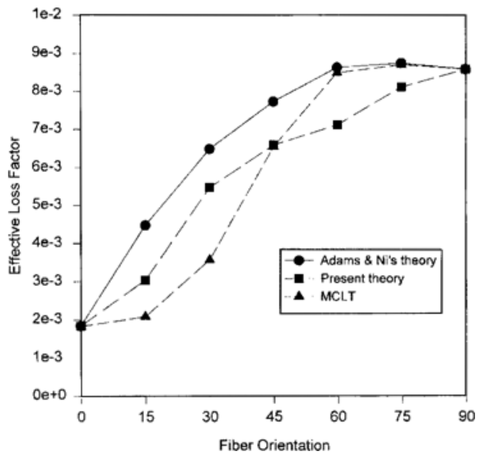


Fig. 2 Comparison of damping loss factors for theories in the $[\theta/-\theta]$s carbon/epoxy laminates as a function of fiber orientation.

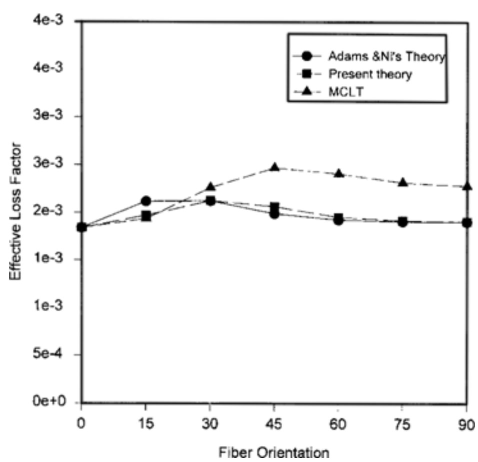


Fig. 3 Comparison of damping loss factors for theories in the $[0/\theta]$s carbon/epoxy laminates as a function of fiber orientation.

occurs in the range of 30° to 60°. These discrepancies may occur owing to their different assumptions and the absence of an accurate expression of the basic Poisson's damping in the modified classical laminate theory. Increasing the percentage of 0° plies in the laminates reduces damping. This observation is consistent even if the viscoelastic response of the matrix is the major damping mechanism. The numerical results demonstrate that damping is significantly influenced by stacking sequence in composites. It is also observed that the fiber orientation with the high-

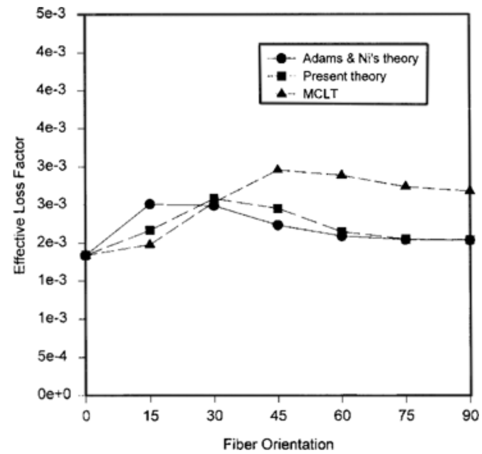


Fig. 4 Comparison of damping loss factors for theories in the $[0/\theta/-\theta]$s carbon/epoxy laminates as a function of fiber orientation.

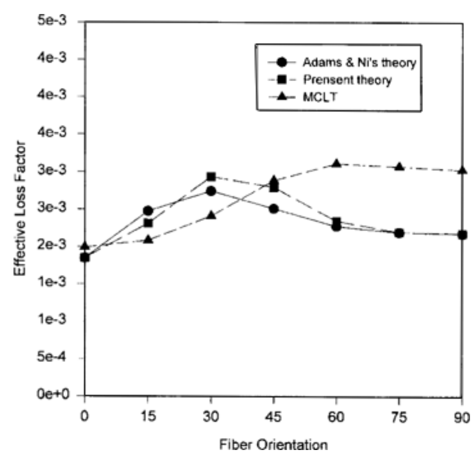


Fig. 5 Comparison of damping loss factors for theories in the $[0/\theta/-\theta/90]$s carbon/epoxy laminates as a function of fiber orientation.

est loss factor must be located near the surface of the laminate to produce the highest loss factor in laminated composites.

4. Conclusions

Three damping prediction models have been compared for symmetric balanced laminated composite beams. The comparisons show that the results of the three models are in reasonable agreement. Also, each theory is valid only for the fundamental flexural mode shape. Damping is highly sensitive to fiber orientation and stacking sequence in each model. The principal curvature approach in strain energy with a linear ply strain condition can provide more accurale solutions than the principal moment approach with ply stress condition in damping prediction of symmetric balanced laminated composites owing to the discontinuous ply stress distribution between laminae. Development of a closed form solution of the basic shear damping and Poisson's damping will help to make more accurate prediction of damping and to reduce discrepancies among the models.

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